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ABSTRACT

The purpose of this study was to formulate a linear programming model to simulate a foundation type support program and to apply this model to a state support program for the public elementary and secondary school districts in the State of Iowa. The model was successful in producing optimal solutions to five objective functions proposed for testing it, and thus it is concluded that the use of a linear programming model to simulate a foundation type state program is indeed feasible. Tables present the study data, and an appendix provides the algebraic matrix used for the solution to each of the five problems. (Author/DB)

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A LINEAR PROGRAMMING MODEL TO OPTIMIZE VARIOUS OBJECTIVE FUNCTIONS OF A FOUNDATION TYPE STATE SUPPORT PROGRAM

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A LINEAR PROGRAMMING MODEL TO OPTIMIZE VARIOUS OBJECTIVE FUNCTIONS OF A FOUNDATION TYPE STATE SUPPORT PROGRAM

Education is the most important and the largest undertaking in the United States. This is true if we think of education as an investment in developing our human resources. The future of this nation will be largely determined by how we invest our financial resources in education today and in the future. The problem of how to finance our investment in education is a matter of major concern throughout the nation because of (1) the rising cost of education, (2) the increasing awareness of the need for quality and equal educational opportunity, and (3) the growing demands upon local property valuations to pay for increasing local governmental services including education.

The expenditures of public elementary and secondary education for all programs operated by public school systems, interest, and capital outlay reached a new high of 39.5 billion in 1969-70, up 10.4 percent from 35.8 billion in 1968-69. This rate of increase is 4.3 percentage points higher than the increase of 6.1 percent in the nation's gross national product (Lee,1970:36).

The rise in educational expenditures may be traced to increased enrollments, inflation, and additions to educational system for quality. Nationally, school enrollments

have increased by 28.5 percent to 59.1 million since 1961 (Lee,1970:5). During the same period the enrollments in Iowa public schools has increased from 589,499 students in 1961 to 659,888 in 1969-70 or an increase of 12 percent.

The loss or buying power of the dollar, due to a rise in prices, or inflation, amounts to approximately 40 percent. This rise alone would add approximately 99 million dollars to the expenditures of 1959-60. It is difficult to keep this rise in perspective because included is a demand for more and better quality services.

The wealth of the district, defined as the assessed valuation per student, determines the amount of revenue received by the district for each mill levied by the local property tax. During 1969-70, in Iowa, the district with highest assessed valuation per pupil was able to raise $31.84 for each mill levied compared to $4.96 in the district with lowest assessed valuation. The poorest district in assessed valuation must be willing to levy a property tax that is approximately six and three tenths times the rate of the wealthiest district in order to be able to provide its students with equal expenditures. Studies have indicated that districts which have low assessed valuation per pupil tend to have higher millage rates but lower expenditures per pupil, offer more units at the secondary level, are larger in size, and have larger average class size (Johnson, 1970: Grabinski, 1970). These inequalities promoted by the present

state support system indicate a definite need to modify the existing program.

The differences in the educational opportunities offered to students living in different areas of the state or nation have been considered by various writers. The responsibility of the state for equality in education may be found in the Supreme Court's specific interpretation of the equal-protection clause in Brown vs. Board of Education as stated by Wise (1968:7). "The opportunity of an education... where the state has undertaken to provide it, is a right which must be available to all on equal terms." The State Constitution of Iowa, Article IX, Section 12, originally provided for education of the youth by the state through a system of common schools.

The level of per pupil expenditure is a valid indicator of the quality of education the student receives. Mort and Vincent (1964:97) support this view by stating: "Three hundred factors have been studied for their effect on schools; of these the amount of money which a school district has to spend...is the most important single factor."

American education has succeeded in providing an education for all, but as stated by Keppel (1966:30) the "necessary revolution" in American education is the end of the "contradiction in practice between quality of education and equality of educational opportunity. The attempt to provide equality of educational opportunity is showing the need for quality in the educational system." Self interest of

influential parents and the state legislatures has impeded progress toward equality (Benson,1965). Coons, Clune & Sugarman (1970:203) agree with Benson that: "The distribution of political power probably makes straight forward legislative adoption of a true power equalizing policy (or any other equality system) impossible at best." Wise (1968:7) believes "The revolution in equality cannot be consumated without aid from the judiciary." These feelings have been strengthened by the recent court decisions in California, Texas and Minnesota.

The present demand placed upon local taxing capabilities by all county and local governments has caused concern throughout the nation. Limited resources from local taxation indicate a need to allocate those funds which are available in order to maximize or minimize a certain pre-determined objective.

The ever increasing size of the educational systems and the economic conditions which exist in this nation have caused taxpayers to become concerned about the future direction of our educational system. The increasing number of bond issue rejections may serve as a guide to the attitude of the taxpayer toward increasing costs.

The attitude that education is an investment for the future is widely proclaimed by educators and is generally accepted by the lay public, however, taxpayers are complaining that taxes cannot continue to rise. These complaints

are generally voiced about local property taxes. Individuals as well as pressure groups are suggesting a greater proportion of educational expenditures be derived from non-property tax sources.

The problem of increasing costs for education will probably become greater in the future (Grabinski, 1970). If additional funds do not become available, educators will have the same problem which faces business and industry, that of allocating fixed resources which are available in a manner which will maximize or minimize predetermined objectives of the educational system. Mathematical programming is one technique which is being widely used in business and industry to achieve maximum utilization of resources.

Decisions, involving the allocation of funds among the many programs, districts, etc., of an educational system, all require the maximum utilization of these resources. These resources are usually fixed, ie., the funds available will not meet all the requests from the competing activities. The importance of these decisions, which will affect the educational opportunities of all students within the system, makes the use of mathematical programming by the educational decision-maker an important tool.

Bruno's (1968) model of a foundation support program for California Junior Colleges demonstrates the effective

use of mathematical programming in solving resource allocation problems in education. He recommends the use of this technique for the allocation of state funds to elementary and secondary public schools.

A review of the literature indicates that foundation type state support programs have been studied extensively since 1925. These studies have used many alternate methods of determining local and state shares of support and the foundation level. Is it possible to build a linear programming model to simulate a foundation type state support program for financing public elementary and secondary education? The large number of variables found in a state public elementary and secondary educational system presents a challenge which has not been attempted by researchers.

The purpose of this study was to formulate a linear programming model to simulate a foundation type support program and to apply this model to a state support program for the public elementary and secondary school districts in the State of Iowa.

The linear programming model was formulated to simulate the fiscal characteristics of the local school districts and their relationships to possible alternatives for funding the entire system. The model was tested using the data taken from the Secretary's Annual Report, 1969-70, for the State of Iowa. The model was solved using the following objective functions:

1. The minimization of the state mandated local qualifying tax rate using the total of 1969-70 state aid funds provided by the state legislature with an objectively determined foundation level.

2. The minimization of state aid costs of the foundation program using a parameterized mandatory local qualifying tax rate of 20 to 40 mills with five mill increments. The foundation level was objectively determined.

3. The minimization of the state mandated local qualifying tax rate requiring a 40% state share of the total foundation program costs. An objectively determined foundation level was used.

4. The minimization of the state mandated local qualifying tax rate using the total of 1969-70 state aid and adding the excess funds raised by the tax rate to the state aid funds for distribution. The foundation level was objectively determined.

5. The maximization of the foundation level using the total of 1969-70 Iowa state aid funds and a parameterized mandatory local qualifying tax rate of 20 to 40 mills with five mill increments.

Formulation of the Generalized Linear Programming Model for the Foundation State Support Program

The variables and the interaction of those variables must be stated mathematically in order to formulate the general linear programming model for the foundation type support program.

On the local district level the per pupil fiscal relationships for district i may be stated:

1. $A_iX + Y_i = F + E_i$

where A_i = The assessed valuation per pupil in average daily membership (ADM) in district i.

X = The uniform state mandated local tax rate expressed in mills for the state.

Y_i = The total share of state aid per pupil in ADM to district i.

F = The foundation level per ADM for the state.

i = 1,2,3, . . ., N, where N equals the number of districts in the state.

E_i = Excess funds raised above the foundation level per pupil/ADM.

Expenditures above the foundation level were not considered in the linear programming model formulated for this study. For the purpose of this study, all districts were considered to be spending at the foundation level. Excess funds above the foundation level which may be raised by the tax rate were computed for each district and totaled for the state.

The above equation (1) may be set to zero in the following manner:

2. $A_iX + Y_i - E_i - F = 0$

This equation was used in this study to indicate the local and state per pupil (ADM) fiscal relationships. If the product of assessed valuation per pupil in average daily membership and the state mandated local tax rate (A_iX) is greater than the foundation level (F), the district (i) will not receive state aid (Y_i). The excess funds (E_i) indicates the amount per pupil the local property tax will raise above the foundation level.

The interaction of the above variables at the local and state level was expressed as follows:

3. $\Sigma ADM_i A_i X - \Sigma ADM_i E_i = L$

where L = The total local funds raised by the state mandated local tax rate up to the foundation level.

The product of the average daily membership, the assessed valuation per pupil and the state mandated local tax rate summed over all the districts of the state ($\Sigma ADM_i A_i X_i$) equals the total local funds raised by the mandated tax rate. The product of the average daily membership and the amount of excess funds raised in those districts, where the tax rate raised more funds than the foundation level, summed over all such districts ($\Sigma ADM_i E_i$) equals the total amount of local funds raised by the state mandated tax rate.

4. $\Sigma ADM_i Y_i = S$

where S = The total state funds (state aid) available.

The product of the average daily membership in district i and the amount of state aid per pupil allocated to the district, summed over all such districts, calculated the total amount of funds required by the state to finance the foundation support program.

The overall costs of the program were expressed as follows:

5. $\Sigma ADM_i A_i X + \Sigma ADM_i Y_i - \Sigma ADM_i E_i = S + L$

where S + L = The total funds, local and state, used in the system.

The sum of total local costs ($\Sigma ADM_i A_i X$) and the total state aid costs ($\Sigma ADM_i Y_i$) minus the total of excess funds ($\Sigma ADM_i E_i$) equals the total amount of funds (local and state) needed to finance the foundation type support program.

The relationships between the total of state aid funds and the total costs of the program were expressed as follows:

6. $S \leq \alpha_1(S + L)$

7. $S \geq \alpha_2(S + L)$

where α_1 = A maximum percentage of total funds allocated to state costs in the system.

α_2 = A minimum percentage of total funds allocated to state costs in the system.

The educational decision maker may want to specify a percent of the total funds needed to finance the foundation type support program. A maximum percentage (α_1) and a minimum percentage (α_2) may be equal if a single percent is necessary. By setting α_1 greater than α_2, a range of percentage was specified. If the percentage of state funds to total costs falls within this range, the constraint is satisfied.

The relationship between the total local funds and the total costs of the program was expressed as:

8. $L \leq \beta_1(S + L)$

9. $L \geq \beta_2(S + L)$

where β_1 = A maximum percentage of local funds in the system.

β_2 = A minimum percentage of local funds in the system.

β_1 and β_2 allow the user to define the percentage of local costs to total program costs in the same fashion as α_1 and α_2 define state costs.

Incorporating the following formulas into the model provided for the computation of the various totals and for future additions to the model.

$A_iX - E_i \geq 0$	Computes the local share of the foundation level for district i.
$\Sigma ADM_iA_iX - \Sigma ADM_i E_i \geq 0$	Computes the total local receipts for the state.
$ADM_iY \geq 0$	Computes the total of state aid funds allocated to district i.
$Max(0, A_iX - F) = E_i$	Computes the amount of excess funds per ADM raised in district i.
$\Sigma ADM_iE_i \geq 0$	Computes the total excess funds raised by the state mandated tax rate (state total).
$\Sigma ADM_iA_iX + \Sigma ADM_iY_i - \Sigma ADM_iE_i \geq 0$	Computes the total overall (state plus local) cost of the program (assumes foundation level spending by each district).

Summary of Constraint Set for Foundation Type State Support Model

In summary, the following equations and inequalities are the constraint set for the linear programming model.

District Constraints (per ADM)	$A_iX + Y_i - E_i - F = 0$

Overall System Constraints

$$\Sigma ADM_i A_i X + \Sigma ADM_i Y_i - \Sigma ADM_i E_i - S - L = 0$$

$$\Sigma ADM_i A_i X - \Sigma ADM_i E_i - L = 0$$

$$\Sigma ADM_i A_i Y_i - S = 0$$

Variable Interaction Constraints

$$S - \alpha_1 (S + L) \leq 0$$

$$S - \alpha_2 (S + L) \geq 0$$

$$L - \beta_1 (S + L) \leq 0$$

$$L - \beta_2 (S + L) \geq 0$$

Computational Formulas

$$A_i X - E_i \geq 0$$

$$\Sigma ADM_i A_i X - \Sigma ADM_i E_i \geq 0$$

$$ADM_i Y_i \geq 0$$

$$Max(0, A_i X - F) = E_i$$

$$\Sigma ADM_i E_i \geq 0$$

$$\Sigma ADM_i A_i X + \Sigma ADM_i Y_i - \Sigma ADM_i E_i \geq 0$$

Summary of Symbols Used in the Model

ADM_i = Average daily membership in district i.

A_i = Assessed valuation per pupil in ADM in district i.

X = Uniform state mandated local tax rate expressed in mills for the state.

Y_i = Total share of state aid per pupil in ADM in district i.

F = Foundation level per pupil in ADM for the state.

i = 1, 2, 3 . . ., N, where N equals the number of districts in the state.

E_i = Excess funds raised by the state mandated tax above the foundation level in district i.

L = Total local funds raised by the state mandated local tax up to the foundation level.

S = Total state funds available.

α_1 = A maximum percentage of total funds allocated to state costs.

α_2 = A minimum percentage of total funds allocated to state costs.

β_1 = A maximum percentage of total funds allocated to local costs.

β_2 = A minimum percentage of total funds allocated to local costs.

Description of Computer Program Used in the Solution of the Model

The computer program which was used for the solution of the linear programming model of a foundation type state support program was the Linear and Separable Programming portion of the IBM/360 Mathematical Programming System. The method used by this system to solve a linear programming problem is a modified simplex algorithm.

The simplex algorithm is based upon a matrix consisting of constraints (or rows) and variables (columns or vectors) which must be linearly independent. All variables must be nonnegative. The bounding feature allows the user to specify a certain range or level for any or all the variables. This allows the user to restrict the value of the coefficients of a column which leads to greater economy in computing time.

An optimal solution is a solution which satisfies all the criteria of the problem and which produces a minimum or maximum value for the object function. An infeasible solution either has variables that have negative values or the value of a variable is outside a specified range. If a feasible solution is found and the constraint rows do not confine the value of the object function to a finite value, the problem is said to be unbounded.

Description of District Inputs to the Model

The inputs to the linear programming model consisted of data supplied by the Secretary's Annual Reports, 1969-70, from each local school district and the Iowa State Department of Education.

Table One contains summary information gathered from the Secretary's Annual Report. The districts have been ranked according to assessed valuation per pupil in average daily membership. The seven districts with the lowest assessed valuation per pupil in ADM, nine districts including the median district, and the seven districts of highest assessed valuation per pupil in ADM are included in this table.

Table One contains the following information:

Number - The rank of the district in assessed valuation per pupil in average daily membership from lowest to highest.

District Number - A six digit number identifying the district.

TABLE 1

IOWA SCHOOL DISTRICT DATA 1969-70*

NUMBER	DISTRICT NUMBER	GENERAL FUND TAX RATE	DISTRICT AVERAGE DAILY MEMBERSHIP	REIMBURSABLE EXPENDITURES PER ADM	ASSESSED VALUATION PER ADM	DISTRICT WEALTH RATIO LOW TO HIGH
1	781476	66.471	15239.2	618.37	4961.21	6.31880
2	574086	75.217	2883.3	753.05	5703.38	5.49655
3	914797	69.972	1039.6	679.96	5728.80	5.47216
4	776579	82.276	3442.9	766.25	5903.19	5.31050
5	71044	60.215	7511.7	663.78	6212.27	5.04629
6	905049	68.233	7975.4	672.02	6292.21	4.98218
7	224095	63.629	511.4	666.36	6392.14	4.90429
223	752988	40.058	653.4	906.33	13126.48	2.38822
224	802602	51.168	323.9	856.56	13142.29	2.38535
225	542943	45.590	340.1	744.41	13146.90	2.38451
226	161926	40.891	936.4	801.88	13157.09	2.38266
227	676987	36.024	1157.6	747.90	13173.95	2.37961
228	571337	53.353	2606.1	900.22	13180.57	2.37842
229	134023	37.465	971.0	762.85	13207.22	2.37362
230	446700	55.170	812.5	1109.63	13226.01	2.37025
231	163691	41.408	768.0	824.62	13351.23	2.34802
447	140999	19.455	1310.1	753.53	27641.10	1.13414
448	133915	33.060	304.4	1016.80	27745.84	1.12986
449	350916	36.960	441.1	1222.93	27995.81	1.11977
450	975877	36.254	621.5	752.60	29506.34	1.06245
451	553897	32.618	248.3	1053.91	30831.75	1.01677
452	762277	28.159	287.1	990.75	31273.54	1.00241
453	755486	25.891	528.6	908.08	31348.91	1.00000

*Source: Iowa, State Department of Education, Secretary's Annual Reports, 1969-70.

General Fund Tax Rate - The 1969-70 general fund tax rate of the district.

District Average Daily Membership - The 1969-70 average daily membership of the district.

Reimbursable Expenditures per ADM - The 1969-70 reimbursable expenditures per pupil in average daily membership of the district. This does not include building fund expenditures.

Assessed Valuation per ADM - The 1969-70 assessed valuation per pupil in average daily membership of the district.

District Wealth Ratio - The 1969-70 wealth ratio was determined by dividing the assessed valuation per ADM of the most wealthy district by the assessed valuation per ADM of each district in the state.

The data for Table One was provided by the State Department of Education on magnetic tape. This tape contained data which was compiled by the local district in the form of the Secretary's Annual Report for 1969-70. A program written in FORTRAN IV by the author was used to read the necessary fields on the tape and write this data and the results of the computations on another tape. The districts were listed by county and district number. This tape was then used as the input for the IBM System/360 - SORT/MERGE program which sorted the districts in ascending order according to assessed valuation per ADM. The results of the SORT program were output on tape to form the input of another program written by the author to determine the local ability ratios.

In addition to Table One, the following data was computed from the Secretary's Annual Reports for 1969-70:

Number of school districts in the State of Iowa	453
Total assessed valuation of the state	$7,279,054,631.
Total average daily membership in the state	654,372.2
Mean assessed valuation per pupil in ADM	$ 11,123.72
Total general fund reimbursable expenditures	$514,010,232.53
Mean general fund reimbursable expenditures per ADM	785.50
Total professional instructional costs	$319,261,919.40
Ratio of professional instructional costs to total reimbursable expenditures	.6211
Total state aid	$112,000,000.00
Total aid from income tax	$ 37,405,552.94

The input data required by the model was a district identification number, the assessed valuation of the district, and the average daily membership of the district. In this study the districts were ranked in relation to assessed valuation per ADM of the district and this rank was used to identify each district.

Objectively Determined Foundation Level

The level of support upon which the foundation program is based was computed for the following problems to be solved in this study.

1. Minimization of state mandated local qualifying tax rate.

2. Minimization of state costs.

3. Minimization of state mandated local qualifying tax rate requiring 40% state share of total costs.

4. Minimization of state mandated local qualifying tax rate with the addition of excess funds to state aid funds.

An objectively determined level of reimbursable expenditures for the State of Iowa was computed using the guidelines stated by the Educational Policies Commission of the National Education Association (1959) and modified as suggested by Jones (1966:25) for state wide applications.

1. The salary to be paid teachers is the most critical factor in the determination of any foundation amount. The median salary paid teachers in the state can be used as a starting point in a foundation formula.

The median salary of teachers in Iowa was determined from data secured from the State Department of Education, which was supplied on magnetic tape. This tape contained data from the Iowa Public School Employees Data Sheets for the year 1969-70.

Salaries for 35.520 professional employees were sorted by the IBM/360 - SORT/MERGE program into asending order. The median teachers (professional instructors) salary for the State of Iowa in 1969-70 was $8,500.

2. A per pupil amount for professional instructional services can be determined by adopting a ratio of a stipulated number of pupils per professional staff member (Jones,1966:25).

The 20:1 ratio suggested by the Educational Policies Commission (1959) was used for this study. Using the median teacher salary for the State of Iowa of $8,500, the per pupil professional instructional cost was $425, (8500 ÷ 20).

> 3. A per pupil amount for all current expenditures can be derived from the cost per pupil for instructional services, using the ratio existing between teachers salaries and total current expenditures. (Jones, 1966:26)

The ratio used in this study is not an estimate. The data for determining the ratio was supplied by the State Department of Education on magnetic tape which contained each district's Secretary's Annual Report for 1969-70.

Average daily membership was used as a basis for determining per pupil computations not only because of its current use in the support program, but because it gives a fair indication of the size of the district. The state total of average daily membership in 1969-70 for the State of Iowa was 654,372.2.

The state total of reimbursable expenditures was used to determine the ratio because those expenditures are now the basis for the present state aid program in the State of Iowa. Expenditures which are covered by federal aid or included under the present special state aid statutes are not included in the reimbursable expenditures. Each district's reimbursable expenditures were computed by adding the district tax receipts, county basic tax receipts, share of income tax receipts, and state aid receipts. The state

total of reimbursable expenditures for 1969-70, as recorded on the Secretary's Annual Report, was $514,010,232.53. The state reimbursable expenditures per pupil based upon 654,372.2 pupils in average daily membership (ADM) were computed to be $758.50 (514,010,232.53 ÷ 654,372.2).

The state total of professional instruction costs was computed by summing each district's professional instruction costs. The district professional instruction costs were computed from the Secretary's Annual Report data by summing the total salaries in each of the following categories; principals, guidance and counseling, substitute teachers, special teachers, librarians, phychological services personnel, audio-visual personnel, and television instruction personnel. The state total of professional instruction costs for 1969-70 was $319,261,919.40. The state professional instruction costs per pupil in average daily membership (ADM) were $487.89 (319,261,919.40 ÷ 654,372.2).

The ratio of state professional instruction costs per pupil to the state reimbursable expenditures per pupil was computed by dividing the state professional instruction costs per pupil ($487.89) by the state reimbursable expenditures per pupil ($785.50). The ratio was .6211 or 62.11%.

Solutions of the Model

A brief summary of the results of each of the optimization problems will be reported. Appendix A contains the algebraic matrix used for the solution to each problem.

Summary of Optimal Solution to Problem 1

1. The minimization of the state mandated local qualifying tax rate using the total of 1969-70 state aid funds provided by the state legislature with an objectively determined foundation level.

For the solution to this problem the foundation level was set at $684.27. The total state costs were set at $149,405,552.94 as the level of expenditures which must be met by the solution. The state summary is reported in Table 2. Table 3 reports the selected local district summary.

A total of 569,088.6 or 86.97 percent of the pupils in average daily membership will receive state aid funds based on this solution. A total of 85,283.6 pupils will not be eligible for state aid funds.

Summary of Optimal Solution to Problem 2

2. The minimization of state aid costs of the foundation program using a parameterized mandatory local qualifying tax rate of 20 to 40 mills with five mill increments. The foundation level was objectively determined.

The foundation level for each of the five parts of the solution was set at $684.27. The state mandated tax rate was set at 20 mills and incremented by five mills up to 40 mills. The state summary is reported in Table 4. Table 5 reports the selected local district summary.

Based on the solution with a state mandated local qualifying tax rate set at 20 mills, all pupils in the State of Iowa will receive state aid funds.

TABLE 2

OPTIMAL SOLUTION -- PROBLEM 1

MINIMIZE STATE MANDATED LOCAL TAX RATE

STATE SUMMARY

STATE MANDATED LOCAL QUALIFYING TAX RATE	42.888
FOUNDATION LEVEL -- PER PUPIL	$ 684.27
TOTAL LOCAL COSTS	298,361,712.35
TOTAL STATE COSTS	149,405,552.94
TOTAL FOUNDATION PROGRAM COSTS	447,767,265.29
TOTAL EXCESS DISTRICT FUNDS	13,821,656.32

TABLE 3

OPTIMAL SOLUTION -- PROBLEM 1

MINIMIZE STATE MANDATED LOCAL TAX RATE

LOCAL DISTRICT SUMMARY
(PER PUPIL)

DISTRICT NUMBER	LOCAL DISTRICT COSTS	STATE COSTS	EXCESS DISTRICT FUNDS
1	$212.78	$471.49	$ 0.00
227	565.00	119.27	0.00
304	677.44	6.83	0.00
453	684.27	0.00	660.22

TABLE 4

OPTIMAL SOLUTION -- PROBLEM 2

MINIMIZE STATE AID COSTS
(INCREMENTED TAX RATE)

STATE SUMMARY

TAX RATE	FOUNDATION LEVEL PER PUPIL	TOTAL LOCAL COSTS	TOTAL STATE COSTS	TOTAL FOUNDATION COSTS	TOTAL EXCESS FUNDS
20.00	$684.27	$145,581,093.07	$302,186,172.22	$447,767,265.29	$ 0.00
25.00	684.27	181,821,120.62	265,946,144.67	447,767,265.29	155,245.72
30.00	684.27	216,837,516.47	230,929,748.82	447,767,265.29	1,534,123.13
35.00	684.27	250,406,163.68	197,361,101.61	447,767,265.29	4,360,749.18
40.00	684.27	281,597,797.74	166,169,467.55	447,767,265.29	9,564,388.39

TABLE 5

OPTIMAL SOLUTION -- PROBLEM 2

MINIMIZE STATE AID COSTS
(INCREMENTED TAX RATE)

LOCAL DISTRICT SUMMARY
(PER PUPIL)

	20 MILLS			25 MILLS			30 MILLS		
DIST. NO.	LOCAL DISTRICT COSTS	STATE COSTS	EXCESS DISTRICT FUNDS	LOCAL DISTRICT COSTS	STATE COSTS	EXCESS DISTRICT FUNDS	LOCAL DISTRICT COSTS	STATE COSTS	EXCESS DISTRICT FUNDS
1	$ 99.22	$585.05	$ 0.00	$124.03	$560.24	$ 0.00	$148.84	$535.43	$ 0.00
227	263.48	420.79	0.00	329.35	354.92	0.00	395.22	289.05	0.00
423							676.66	7.61	0.00
445				681.74	2.53	0.00			
453	626.98	57.29	0.00						
453	626.98	57.29	0.00	684.27	0.00	99.45	684.27	0.00	256.20

	35 MILLS			40MILLS		
DIST. NO.	LOCAL DISTRICT COSTS	STATE COSTS	EXCESS DISTRICT FUNDS	LOCAL DISTRICT COSTS	STATE COSTS	EXCESS DISTRICT FUNDS
1	$173.64	$510.63	$ 0.00	$198.45	$485.82	$ 0.00
227	461.09	223.18	0.00	526.96	157.31	0.00
339				683.11	1.16	0.00
392	681.50	2.77	0.00			
453	684.27	0.00	412.94	684.27	0.00	569.69

A total of 650,165.8 or 99.36 percent of the pupils will receive state aid funds when the state mandated local qualifying tax rate was incremented to 25 mills. A total of 4,206.4 pupils will not be eligible for state aid funds.

When the state mandated local qualifying tax rate was set at 30 mills, a total of 638,296.7 or 97.56 percent of the pupils in ADM will receive state aid funds. A total of 16,075.5 pupils will not be eligible for state aid funds.

With the state mandated local qualifying tax rate set at 35 mills, a total of 622,888.8 or 95.19 percent of the pupils will receive state aid funds, with 31,483.4 pupils not receiveing those funds.

A total of 591,486.6 or 90.39 percent of the pupils will receive state aid funds when the state mandated local qualifying tax rate is set at 40 mills. A total of 62,885.6 pupils will not be eligible for state aid funds.

Summary of Optimal Solution to Problem 3

3. The minimization of the state mandated local qualifying tax rate requiring a 40% state share of the total foundation program costs. An objectively determined foundation level was used.

The total state costs were constrained in this solution to be 40 percent of the total foundation program costs. The foundation level was set at $684.27. The state summary is reported in Table 6. Table 7 reports the selected local district summary.

TABLE 6

OPTIMAL SOLUTION -- PROBLEM 3

MINIMIZE STATE MANDATED LOCAL TAX RATE
(STATE COST 40% OF TOTAL COSTS)

STATE SUMMARY

STATE MANDATED LOCAL QUALIFYING TAX RATE	37.869
FOUNDATION LEVEL -- PER PUPIL	$ 684.27
TOTAL LOCAL COSTS	268,660,359.17
TOTAL STATE COSTS	179,106,906.12
TOTAL FOUNDATION PROGRAM COSTS	447,767,265.29
TOTAL EXCESS DISTRICT FUNDS	6,993,543.10

TABLE 7

OPTIMAL SOLUTION -- PROBLEM 3

MINIMIZE STATE MANDATED LOCAL TAX RATE
(STATE COST 40% OF TOTAL COSTS)

LOCAL DISTRICT SUMMARY
(PER PUPIL)

DISTRICT NUMBER	LOCAL DISTRICT COSTS	STATE COSTS	EXCESS DISTRICT FUNDS
1	$187.88	$496.39	$ 0.00
227	498.89	185.38	0.00
362	683.51	0.76	0.00
453	684.27	0.00	502.90

A total of 604,309.6 or 92.35 percent of the pupils will receive state aid funds based upon this solution. A total of 50,062.6 pupils will not be eligible for state aid funds.

Summary of Optimal Solution to Problem 4

4. The minimization of the state mandated local qualifying tax rate using the total of 1969-70 state aid and adding the excess funds raised by the tax rate to the state aid funds for distribution. The foundation level was objectively determined.

The excess district funds are those funds raised by the state mandated local qualifying tax rate above the foundation level of $684.27. These funds are added to the state funds available to the foundation program to be distributed to other less wealthy districts. The total state costs, not including the excess district funds, were set at $149,405,552.94. The state summary is reported in Table 8. Table 9 reports the selected district summary.

A total of 587,617.1 or 89.80 percent of the pupils in average daily membership will receive state aid based upon this solution. A total of 66,755.1 pupils will not be eligible for state aid funds.

Summary of Optimal Solution to Problem 5

5. The maximization of the foundation level using the total of 1969-70 Iowa state aid funds and a parameterized mandatory local qualifying tax rate of 20 to 40 mills with five mill increments.

TABLE 8

OPTIMAL SOLUTION -- PROBLEM 4

MINIMIZE STATE MANDATED LOCAL TAX RATE
(EXCESS DISTRICT FUNDS ADDED TO STATE COSTS)

STATE SUMMARY

STATE MANDATED LOCAL QUALIFYING TAX RATE	40.989
FOUNDATION LEVEL -- PER PUPIL	$ 684.27
TOTAL LOCAL COSTS	287,467,122.60
TOTAL STATE COSTS	149,405,552.94
TOTAL EXCESS DISTRICT FUNDS	10,894,589.75
TOTAL FOUNDATION PROGRAM COSTS	447,767,265.29

TABLE 9

OPTIMAL SOLUTION -- PROBLEM 4

MINIMIZE STATE MANDATED LOCAL TAX RATE
(EXCESS DISTRICT FUNDS ADDED TO STATE COSTS)

LOCAL DISTRICT SUMMARY
(PER PUPIL)

DISTRICT NUMBER	LOCAL DISTRICT COSTS	STATE COSTS	EXCESS DISTRICT FUNDS
1	$203.36	$480.91	$ 0.00
227	539.99	144.28	0.00
333	683.91	0.36	0.00
453	684.27	0.00	600.69

The state mandated local qualifying tax rate was set at 20 mills and incremented by 5 mills up to 40 mills. The total state costs were set at $149,405,552.94; the level of state costs in 1969-70 for aid to public elementary and secondary education in the State of Iowa. The state summary is reported in Table 10. Table 11 reports the selected local district summary.

Based on the solution with the state mandated local qualifying tax rate set at 20 mills, a total of 637,836.2 or 97.47 percent of the pupils in average daily membership will receive state aid funds. A total of 16,536.0 pupils will not be eligible for state aid funds.

When the state mandated local qualifying tax rate was set at 25 mills, a total of 626,539.9 or 95.75 percent of the pupils in average daily membership will receive state aid funds. A total of 27,832.3 pupils will not be eligible for state aid funds.

A total of 608,626.1 or 93.01 percent of the pupils will receive state aid funds when the state mandated local qualifying tax rate was incremented to 30 mills. A total of 45,746.1 pupils will not be eligible for state aid funds.

With the state mandated local qualifying tax rate set at 35 mills, 594,980.5 or 90.92 percent of the pupils in average daily membership will receive state aid funds. A total of 59,391.7 pupils will not be eligible for state aid funds.

TABLE 10

OPTIMAL SOLUTION -- PROBLEM 5

MAXIMIZE FOUNDATION LEVEL
(INCREMENTED TAX RATE)

STATE SUMMARY

TAX RATE	FOUNDATION LEVEL PER PUPIL	TOTAL LOCAL COSTS	TOTAL STATE COSTS	TOTAL FOUNDATION COSTS	TOTAL EXCESS FUNDS
20.00	$449.05	$144,442,844.95	$149,405,552.94	$293,848,397.89	$ 1,138,248.12
25.00	502.28	179,275,006.54	149,405,552.94	328,680,559.48	2,701,359.79
30.00	554.46	213,415,330.41	149,405,552.94	362,820,883.35	4,956,309.19
35.00	605.49	246,812,089.08	149,405,552.94	396,217,642.02	7,954,823.79
40.00	655.72	279,680,699.92	149,405,552.94	429,086,252.86	11,481,486.22

TABLE 11

OPTIMAL SOLUTION -- PROBLEM 5

MAXIMIZE FOUNDATION LEVEL
(INCREMENTED TAX RATE)

LOCAL DISTRICT SUMMARY
(PER PUPIL)

	20 MILLS			25 MILLS			30 MILLS		
DIST. NO.	LOCAL DISTRICT COSTS	STATE COSTS	EXCESS DISTRICT FUNDS	LOCAL DISTRICT COSTS	STATE COSTS	EXCESS DISTRICT FUNDS	LOCAL DISTRICT COSTS	STATE COSTS	EXCESS DISTRICT FUNDS
1	$ 99.22	$349.83	$ 0.00	$124.03	$378.25	$ 0.00	$148.84	$405.62	$ 0.00
227	263.48	185.57	0.00	329.35	172.93	0.00	395.22	159.24	0.00
369							551.76	2.70	0.00
399				501.55	0.73	0.00			
422	444.70	4.35	0.00						
453	449.05	0.00	177.92	502.28	0.00	281.44	554.46	0.00	386.01

	35 MILLS			40 MILLS		
DIST. NO.	LOCAL DISTRICT COSTS	STATE COSTS	EXCESS DISTRICT FUNDS	LOCAL DISTRICT COSTS	STATE COSTS	EXCESS DISTRICT FUNDS
1	$173.64	$431.85	$ 0.00	$198.45	$457.27	$ 0.00
227	461.09	144.40	0.00	526.96	128.76	0.00
319				655.57	0.15	0.00
345	604.93	0.57	0.00			
453	605.49	0.00	491.72	655.72	0.00	598.23

Incrementing the state mandated local qualifying tax to 40 mills will allow 579,971.7 or 86.97 percent of the pupils in average daily membership to receive state aid funds. A total of 74,400.5 pupils will not be eligible for state aid funds.

Conclusions and Recommendations

The linear programming model, formulated to simulate a foundation type state support program for financing elementary and secondary education, was successful in producing optimal solutions to the five objective functions proposed for testing the model. As a result of the optimal solutions produced by the model, the use of a linear programming model to simulate a foundation type state support program is indeed feasible.

The model was capable of simulating the financial characteristics in terms of the number of pupils in average daily membership of each of the 453 local school districts in the State of Iowa. In addition to the local characteristics, the state contribution to the foundation program was set at $149,405,552.92, which was the level of state aid to public school districts in 1969-70 in the State of Iowa, for three of the five objective functions solved by the model.

The flexibility of the model was tested by solving five different problems, which were selected as possible

variations in the distribution of funds to a foundation type program. The change of objective function required only minor changes in the generalized model. The changes in the generalized model are noted in the Summary of Solutions to the Optimization Problems.

The use of mathematical techniques, other than linear programming, could have been used to solve the problems presented in this study. Computer programs may be written to solve each specific problem. The solution to Problem 2 may be calculated, using a desk calculator. If only one problem is to be solved and other possible alternatives are not being investigated, the use of other techniques may save both time and effort for the educational decision-maker. If the solutions of several alternative methods of distribution of state funds are to be investigated, the use of the linear programming model will enable the investigator to calcualte the solution of each problem with minor changes in the generalized model. The ability, to provide without considerable delay a solution to each of the alternatives suggested by the educational decision-makers, gives the user of a linear programming model a decided advantage in terms of time and cost.

The arbitrary selection of the five problems which were optimized in this study indicates to the decision-maker the variety of possible methods of distributing funds for a foundation type state support program. The priorities of

the educational system will dictate the most acceptable method of distributing the funds available to the state support program. The following are other methods of distribution which could be solved using the model.

1. A minimum mandatory amount of state aid per pupil in average daily membership.

2. A simulation of the financial characteristics of proposed redistricting plans.

3. Using a combination of methods used in the five problems solved in this study, such as, the adding of excess district funds to state costs in Problem 5.

The model could be expanded, taking into consideration the differences between educational programs provided in each school district, the differences in costs of education between rural and urban school districts, and the differences in educational costs among students of different socioeconomic groups. The above recommendations would require additional research to determine the mathematical ratios necessary to simulate each characteristic.

Computer programs have been developed for projecting future educational costs. Using these techniques to predict future financial characteristics of the optimal solutions produced by the model would help to provide for more equitable funding of public elementary and secondary education.

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APPENDIX A

PROBLEM 1

ROW NAME / COLUMN NAME	F	S	L	S+L	X	$\Sigma ADM_i E_i$	E_i	Y_i	$A_i X$	SIGN	RIGHT HAND SIDE
OBJECT					1					N	
F_i	-1							1	1	E	0
$\Sigma ADM_i Y_i$		-1						ADM_i		E	0
$\Sigma ADM_i A_i X$			-1						ADM_i	E	0
S + L		1	1	-1						E	0
$\Sigma ADM_i E_i$						-1	ADM_i			E	0
$A_i X$					A_i		-1		-1	E	0
BOUND F	1									E	$ 684.27
BOUND S		1								E	$149,405,552.94
BOUND $A_i X$									1	L.	$684.27

APPENDIX A (CONTINUED)

PROBLEM 2

ROW NAME \ COLUMN NAME	F	S	L	S+L	X	$\Sigma ADM_i E_i$	E_i	Y_i	$A_i X$	SIGN	RIGHT HAND SIDE	TAX RANGE
OBJECT		1								N		
F_i	-1							1	1	E	0	
$\Sigma ADM_i Y_i$		-1						ADM_i		E	0	
$\Sigma ADM_i A_i X$			-1						-1	E	0	
S + L		1	1	-1						E	0	
$\Sigma ADM_i E_i$						-1	ADM_i			E	0	
$A_i X$					A_i		-1		-1	E	0	
X					1					E	.020	.005
BOUND F	1									E	$684.27	
BOUND $A_i X$									1	L	$684.27	

APPENDIX A (CONTINUED)

PROBLEM 3

ROW NAME / COLUMN NAME	F	S	L	S+L	X	$\Sigma ADM_i E_i$	E_i	Y_i	$A_i X$	SIGN	RIGHT HAND SIDE
OBJECT					1					N	
F_i	-1							1	1	E	0
$\Sigma ADM_i Y_i$		-1						ADM_i		E	0
$\Sigma ADM_i Y_i$			-1						ADM_i	E	0
S + L		1	1	-1						E	0
$\Sigma ADM_i E_i$						-1	ADM_i			E	0
$A_i X$					A_i		-1		-1	E	0
α_1(S+L)		1		-.4						E	0
α_2(S+L)		1		-.4						E	0
β_1(S+L)			1	-.6						E	0
β_2(S+L)			1	-.6						E	0
BOUND F	1									E	\$684.27
BOUND $A_i X$									1	L	\$684.27

APPENDIX A (CONTINUED)

PROBLEM 4

ROW NAME \ COLUMN NAME	F	S	L	$S+L$	X	$\Sigma ADM_i E_i$	E_i	Y_i	$A_i X$	SIGN	RIGHT HAND SIDE
OBJECT					1					N	
F_i	-1							1	1	E	0
$\Sigma ADM_i Y_i$		-1						ADM_i		E	0
$\Sigma ADM_i A_i X$			-1						ADM_i	E	0
$S + L$		1	1	-1		1				E	0
$\Sigma ADM_i E_i$						-1	ADM_i			E	0
$A_i X$					A_i		-1		-1	E	0
BOUND F	1									E	$ 684.27
BOUND S		1								E	$149,405,552.94
BOUND $A_i X$									1	L	$ 684.27

APPENDIX A (CONTINUED)

PROBLEM 5

ROW NAME \ COLUMN NAME	F	S	L	S+L	X	$\Sigma ADM_i E_i$	E_i	Y_i	$A_i X$	SIGN	RIGHT HAND SIDE	TAX RANGE
OBJECT	1									N		
F_i	-1							1	1	E	0	
$\Sigma ADM_i Y_i$		-1						ADM_i		E	0	
$\Sigma ADM_i A_i X$			-1						ADM_i	E	0	
S + L		1	1	-1						E	0	
$\Sigma ADM_i E_i$						-1	ADM_i			E	0	
$A_i X$					A_i		-1		-1	E	0	
X					1					E	.020	.005
BOUND S		1								E	$149,405,552.94	